Ecuații- rezolvați în R ecuațiile:

$x+\sqrt{2}=\sqrt{8} \Leftrightarrow x=2\sqrt{2}-\sqrt{2} \Leftrightarrow x=\sqrt{2} \Rightarrow S=\{2\sqrt{2}\}$

$-3\frac{1}{2}\cdot x-1,5=\sqrt{2}^2 \Leftrightarrow -\frac{7}{2}\cdot x-\frac{3}{2}=2 \mid \cdot 2 \Leftrightarrow$

$-7x-3=4 \Leftrightarrow -7x=4+3 \Leftrightarrow -7x=7 \Leftrightarrow x=\frac{7}{-7} \Leftrightarrow x=-1$

$S=\{-1\}$

$3\frac{1}{2}=\frac{3\cdot 2+1}{2}=\frac{7}{2}$

$-3\frac{1}{2}=-\frac{3\cdot 2+1}{2}=-\frac{7}{2}$

$1,5=\frac{15}{10}=\frac{3}{2}$

$3\frac{1}{3}\cdot x-2\frac{1}{2}=\sqrt{125}:\sqrt{5}$

$\frac{3\cdot 3+1}{3}\cdot x-\frac{2\cdot 2+1}{2}=\sqrt{125}:\sqrt{5}$

$\frac{10}{3}\cdot x-\frac{5}{2}=\sqrt{125:5}$

$\frac{10}{3}\cdot x-\frac{5}{2}=\sqrt{25}$

$\frac{10}{3}\cdot x-\frac{5}{2}=5$

$\frac{10}{3}x=5+\frac{5}{2}$

$\frac{10}{3}x=\frac{10}{2}+\frac{5}{2}$

$\frac{10}{3}x=\frac{15}{2}$

$x=\frac{15}{2}:\frac{10}{3}$

$x=\frac{15}{2}\cdot\frac{3}{10}$

$x=\frac{9}{5}$

$S=\{\frac{9}{5}\}$

$x\cdot\sqrt{5}-\frac{1}{\sqrt{5}}=0 \Leftrightarrow$

$x\cdot\sqrt{5}=0+\frac{1}{\sqrt{5}} \Leftrightarrow$

$x\cdot\sqrt{5}=\frac{1}{\sqrt{5}} \Leftrightarrow$

$x=\frac{1}{\sqrt{5}}:\frac{\sqrt{5}}{1} \Leftrightarrow$

$x=\frac{1}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}} \Rightarrow$

$x=\frac{1}{5} \Rightarrow S=\{\frac{1}{5}\}$

$\frac{x}{\sqrt{3}}+\sqrt{12}=\sqrt{27} \Leftrightarrow$

$\frac{x}{\sqrt{3}}+2\sqrt{3}=3\sqrt{3} \Leftrightarrow$

$\frac{x}{\sqrt{3}}=3\sqrt{3}-2\sqrt{3} \Leftrightarrow$

$\frac{x}{\sqrt{3}}=\sqrt{3} \Leftrightarrow$

$x=\sqrt{3}\cdot\sqrt{3} \Leftrightarrow$

$x=3 \Rightarrow S=\{3\}$

$4,5\cdot x+9=13,3$

$\frac{45}{10}\cdot x+9=\frac{133}{10}$

$\frac{9}{2}\cdot x+9=\frac{27}{2} \mid \cdot 2$

$2\cdot\frac{9}{2}x+18=2\cdot\frac{27}{2}$

$9x+18=27$

$9x=9$

$x=1$

$S=\{1\}$

$-\sqrt{5}\cdot x+10=\sqrt{20}\cdot x-20 \Leftrightarrow$

$-\sqrt{5}\cdot x-\sqrt{20}\cdot x=-20-10 \Leftrightarrow$

$-\sqrt{5}\cdot x-\sqrt{20}\cdot x=-30 \mid \cdot \frac{1}{\sqrt{5}} \Leftrightarrow$

$-\sqrt{5}\cdot x\cdot\frac{1}{\sqrt{5}}-\sqrt{20}\cdot x\cdot\frac{1}{\sqrt{5}}=-30\cdot\frac{1}{\sqrt{5}} \Leftrightarrow$

$-x-\sqrt{4}x=-\frac{30}{\sqrt{5}} \Leftrightarrow$

$-x-2x=-\frac{30\sqrt{5}}{5} \Leftrightarrow$

$-3x=-6\sqrt{5} \Leftrightarrow$

$x=\frac{-6\sqrt{5}}{3} \Rightarrow S=\{-2\sqrt{5}\}$

$\frac{y-\sqrt{2}}{3}+\frac{y+\sqrt{2}}{2}=4\sqrt{2}+y \Leftrightarrow$

$\frac{2y-2\sqrt{2}}{6}+\frac{3y+3\sqrt{2}}{6}=4\sqrt{2}+y \mid \cdot 6 \Leftrightarrow$

$6\cdot\frac{2y-2\sqrt{2}}{6}+6\cdot\frac{3y+3\sqrt{2}}{6}=4\sqrt{2}\cdot 6+y\cdot 6 \Leftrightarrow$

$+2y-2\sqrt{2}+3y+3\sqrt{2}=24\sqrt{2}+6y \Leftrightarrow$

$5y+\sqrt{2}=24\sqrt{2}+6y \Leftrightarrow$

$+5y-6y=24\sqrt{2}-\sqrt{2} \Leftrightarrow$

$-y=23\sqrt{2} \mid \cdot(-1) \Leftrightarrow$

$y=-23\sqrt{2} \Rightarrow S=\{-23\sqrt{2}\}$

$(y-\sqrt{3})\cdot(y-\sqrt{5})=0$

Știm că: $2\cdot 0=0 \quad 0\cdot 2=0 \quad 0\cdot 0=0$

Deci: $a\cdot b=0 \Rightarrow a=0$ SAU $b=0$

$2\cdot 3=6\neq 0$

Deci: $a\cdot b\neq 0 \Rightarrow a\neq 0$ și $b\neq 0$

$y-\sqrt{3}=0$ sau $y-\sqrt{5}=0 \Rightarrow S=\{\sqrt{3};\sqrt{5}\}$

$y=\sqrt{3}$ $\quad y=\sqrt{5}$

$(2x-\sqrt{3})\cdot(3x+\sqrt{6})=0$

$(2x-\sqrt{3})\cdot(3x-\sqrt{6})=0$

$\Downarrow$

$2x-\sqrt{3}=0$ SAU $3x-\sqrt{6}=0$

$2x=\sqrt{3}$ $\quad 3x=\sqrt{6}$

$x=\frac{\sqrt{3}}{2}$ $\quad x=\frac{\sqrt{6}}{3}$

$\Rightarrow S=\left\{\frac{\sqrt{3}}{2};\frac{\sqrt{6}}{3}\right\}$

$$\left|7-\sqrt{7}\cdot x\right|=14$$

Dacă $x\in\mathbb{R}$, $a>0 \Rightarrow |x|=a \Leftrightarrow x=a$ sau $x=-a$

$$|x|=\begin{cases} x, \text{ dacă } x>0 \\ 0, \text{ dacă } x=0 \\ -x, \text{ dacă } x<0 \end{cases}$$

Ex: $|3|=3$, $|0|=0$, $|-3|=-(-3)=3$

$|x|=3 \Rightarrow x=3$ sau $x=-3$

1) $|x|\geq 0$, $(\forall)\, x\in\mathbb{R}$

2) $|x\cdot y|=|x|\cdot|y|$, $(\forall)\, x,y\in\mathbb{R}$

$x=-3$, $y=+2$ $\Rightarrow |-3\cdot 2|=|-6|=6$; $|-3|\cdot|2|=3\cdot 2=6$ $\Rightarrow |-3\cdot 2|=|-3|\cdot|2|$

3) $\left|\frac{x}{y}\right|=|x:y|=|x|:|y|=\frac{|x|}{|y|}$ $(\forall)\, x\in\mathbb{R}$, $y\in\mathbb{R}^*$ $(y\neq 0)$

$x=-12$, $y=-6$ $\Rightarrow \left|\frac{-12}{-6}\right|=|+2|=2$; $\frac{|-12|}{|-6|}=\frac{12}{6}=2$ $\Rightarrow \left|\frac{-12}{6}\right|=\frac{|-12|}{|6|}$

4) Atenție: $|x+y|\neq|x|+|y|$ $|x-y|\neq|x|-|y|$

Corect: $|x+y|\leq|x|+|y|$ (inegalitatea triunghiului)

Ex: $x=-4$, $y=1$ $\Rightarrow |x+y|=|-4+1|=|-3|=3$

$|x|+|y|=|-4|+|1|=4+1=5$

$|-4+1|<|-4|+|1|$

$x=-5$, $y=-2$ $\Rightarrow |x+y|=|-5+(-2)|=|-5-2|=|-7|=7$

$|x|+|y|=|-5|+|-2|=5+2=7$

$|-5+(-2)|=|-5|+|-2|$

$|7-\sqrt{7}\cdot x|=14$

$\Downarrow$

$7-\sqrt{7}\cdot x=14$

$\sqrt{7}\cdot x=14-7$

$\sqrt{7}\cdot x=7$

$x=\frac{7}{\sqrt{7}}$

$x=\frac{7\sqrt{7}}{7}$

$x=\sqrt{7}$

sau $7-\sqrt{7}\cdot x=-14$

$\sqrt{7}\cdot x=-14-7$

$\sqrt{7}\cdot x=-21$

$x=-\frac{21}{\sqrt{7}}$

$x=-\frac{21\sqrt{7}}{7}$

$x=-3\sqrt{7}$

$\Rightarrow S=\{3\sqrt{7},-3\sqrt{7}\}$

$|-2\sqrt{3}+\sqrt{3}\cdot x|=-2$

Stim că $(\forall)\, x\in\mathbb{R}\Rightarrow |-2\sqrt{3}+\sqrt{3}\cdot x|\geq 0$

$-2<0$

(imposibil) $\Rightarrow S=\emptyset$

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