Ecuații în $\mathbb{R}$ - rezolvați în $\mathbb{R}$ ecuațiile

$(a+b)\cdot(a-b)=a^2-b^2$

$(a\pm b)^2=a^2\pm 2ab+b^2$

$(x+3)\cdot(x-3)=16 \Leftrightarrow x^2-3^2=16 \Leftrightarrow$

$x^2-9=16 \Leftrightarrow x^2=16+9 \Leftrightarrow x^2=25 \Leftrightarrow \sqrt{x^2}=\sqrt{25} \Leftrightarrow |x|=5 \Rightarrow$

$x=5$ sau $x=-5 \Rightarrow S=\{5;-5\}$

$\sqrt{A\pm\sqrt{B}}=\sqrt{\frac{A+\sqrt{A^2-B}}{2}}\pm\sqrt{\frac{A-\sqrt{A^2-B}}{2}}$

$(2x+5)\cdot(2x-5)=24 \Leftrightarrow$

$2x^2-5=24$ (GREȘIT!)

$(2x+5)\cdot(2x-5)=24 \Leftrightarrow (2x)^2-5^2=24 \Leftrightarrow 4x^2-25=24 \Leftrightarrow$

$4x^2=24+25 \Leftrightarrow 4x^2=49 \Leftrightarrow x^2=\frac{49}{4} \Leftrightarrow \sqrt{x^2}=\sqrt{\frac{49}{4}} \Leftrightarrow |x|=\frac{\sqrt{49}}{\sqrt{4}} \Leftrightarrow$

$|x|=\frac{7}{2} \Rightarrow \begin{cases} x=\frac{7}{2} \\ \text{sau} \\ x=-\frac{7}{2} \end{cases} \Rightarrow S=\left\{\frac{7}{2};-\frac{7}{2}\right\}$

$(3x-4)\cdot(3x+4)=9 \Leftrightarrow (3x)^2-4^2=9 \Leftrightarrow 9x^2=25 \Rightarrow x^2=\frac{25}{9} \Rightarrow \sqrt{x^2}=\sqrt{\frac{25}{9}} \Rightarrow$

$|x|=\frac{5}{3} \Rightarrow \begin{cases} x=\frac{5}{3} \\ \text{sau} \\ x=-\frac{5}{3} \end{cases} \Rightarrow S=\left\{\frac{5}{3},-\frac{5}{3}\right\}$

$(5x-1)(5x+1)=3 \Leftrightarrow (5x)^2-1^2=3 \Leftrightarrow 25x^2-1=9 \Leftrightarrow 25x^2=10 \Leftrightarrow$

$x^2=\frac{10}{25} \Leftrightarrow \sqrt{x^2}=\sqrt{\frac{10}{25}} \Leftrightarrow |x|=\frac{\sqrt{10}}{\sqrt{25}} \Leftrightarrow |x|=\frac{\sqrt{10}}{5} \Rightarrow \begin{cases} x=\frac{\sqrt{10}}{5} \\ \text{sau} \\ x=-\frac{\sqrt{10}}{5} \end{cases} \Rightarrow$

$S=\left\{\frac{\sqrt{10}}{5},-\frac{\sqrt{10}}{5}\right\}$

$(3x-2)(3x+2)=-5 \Leftrightarrow (3x)^2-2^2=-5 \Leftrightarrow 9x^2-4=-5 \Leftrightarrow 9x^2=-5+4 \Leftrightarrow$

$9x^2=-1 \Rightarrow x^2=-\frac{1}{9}$

$x^2\geq 0, \forall x\in\mathbb{R}$

$-\frac{1}{9}<0$

$\Big\}$ imposibil $\Rightarrow S=\emptyset$

$x^2=4-2\sqrt{3} \Leftrightarrow x^2=4-\sqrt{12} \Leftrightarrow \sqrt{x^2}=\sqrt{4-\sqrt{12}} \Leftrightarrow |x|=\sqrt{4-\sqrt{12}} \Leftrightarrow$

$|x|=\sqrt{\frac{4+\sqrt{4^2-12}}{2}}-\sqrt{\frac{4-\sqrt{4^2-12}}{2}} \Leftrightarrow |x|=\sqrt{\frac{4+\sqrt{4}}{2}}-\sqrt{\frac{4-\sqrt{4}}{2}} \Leftrightarrow |x|=\sqrt{\frac{4+2}{2}}-\sqrt{\frac{4-2}{2}}$

$\Leftrightarrow |x|=\sqrt{3}-\sqrt{1} \Rightarrow \begin{cases} x=\sqrt{3}-1 \\ \text{sau} \\ x=-(\sqrt{3}-1) \end{cases} \Rightarrow \begin{cases} x=\sqrt{3}-1 \\ x=-\sqrt{3}+1 \end{cases} \Rightarrow S=\{\sqrt{3}-1,-\sqrt{3}+1\}$

$x^2 = 7 - 2\sqrt{10} \Leftrightarrow \sqrt{x^2} = \sqrt{7-2\sqrt{10}} \Leftrightarrow |x| = \sqrt{7-2\sqrt{10}} \Leftrightarrow |x| = \sqrt{7-\sqrt{40}}$

$|x| = \sqrt{\frac{7+\sqrt{49-40}}{2}} - \sqrt{\frac{7-\sqrt{49-40}}{2}} \Leftrightarrow |x| = \sqrt{\frac{7+\sqrt{9}}{2}} - \sqrt{\frac{7-\sqrt{9}}{2}} \Leftrightarrow |x| = \sqrt{\frac{10}{2}} - \sqrt{\frac{4}{2}}$

$|x| = \sqrt{5} - \sqrt{2} \Rightarrow \begin{cases} x = \sqrt{5} - \sqrt{2} \\ x = -(\sqrt{5} - \sqrt{2}) \Rightarrow x = -\sqrt{5} + \sqrt{2} \end{cases} \Rightarrow S = \{\sqrt{5} - \sqrt{2}, -\sqrt{5} + \sqrt{2}\}$

$x^2 = 14 + 6\sqrt{5} \Leftrightarrow \sqrt{x^2} = \sqrt{14 + \sqrt{36 \cdot 5}} \Leftrightarrow |x| = \sqrt{14 + \sqrt{180}} \Leftrightarrow$

$|x| = \sqrt{\frac{14+\sqrt{196-180}}{2}} + \sqrt{\frac{14-\sqrt{196-180}}{2}} \Leftrightarrow |x| = \sqrt{\frac{14+\sqrt{16}}{2}} + \sqrt{\frac{14-\sqrt{16}}{2}}$

$|x| = \sqrt{\frac{14+4}{2}} + \sqrt{\frac{14-4}{2}} \Rightarrow |x| = \sqrt{9} + \sqrt{5} \Rightarrow |x| = 3 + \sqrt{5} \Rightarrow \begin{cases} x = 3 + \sqrt{5} \\ x = -3 - \sqrt{5} \end{cases} \Rightarrow$

$S = \{3 + \sqrt{5}, -3 - \sqrt{5}\}$

$x^2 = 8 - 2\sqrt{15} \Leftrightarrow |x| = \sqrt{8 - \sqrt{60}} \Leftrightarrow |x| = \sqrt{\frac{8+\sqrt{64-60}}{2}} - \sqrt{\frac{8-\sqrt{64-60}}{2}} \Leftrightarrow |x| = \sqrt{\frac{8+2}{2}} - \sqrt{\frac{8-2}{2}}$

$|x| = \sqrt{5} - \sqrt{3} \Rightarrow x = \sqrt{5} - \sqrt{3}$ sau $x = -\sqrt{5} + \sqrt{3} \Rightarrow S = \{\sqrt{5} - \sqrt{3}, -\sqrt{5} + \sqrt{3}\}$

$(x-3)^2 = 25 \Leftrightarrow \sqrt{(x-3)^2} = \sqrt{25} \Leftrightarrow |x-3| = 5 \Rightarrow$

$x - 3 = 5 \Rightarrow x = 5 + 3 \Rightarrow x = 8$

sau

$x - 3 = -5 \Rightarrow x = -5 + 3 \Rightarrow x = -2$

$S = \{8; -2\}$

$(2x+1)^2 = 6 \Leftrightarrow \sqrt{(2x+1)^2} = \sqrt{6} \Leftrightarrow |2x+1| = \sqrt{6} \Rightarrow$

$2x + 1 = \sqrt{6} \Rightarrow 2x = \sqrt{6} - 1 \Rightarrow x = \frac{\sqrt{6}-1}{2}$

sau

$2x + 1 = -\sqrt{6} \Rightarrow 2x = -\sqrt{6} - 1 \Rightarrow x = \frac{-\sqrt{6}-1}{2} \Rightarrow x = -\frac{\sqrt{6}+1}{2}$

$S = \{\frac{\sqrt{6}-1}{2}, -\frac{\sqrt{6}+1}{2}\}$

$(3x-1)^2 = 25 \Leftrightarrow \sqrt{(3x-1)^2} = \sqrt{25} \Leftrightarrow |3x-1| = 5 \Rightarrow$

$3x - 1 = 5 \Rightarrow 3x = 6 \Rightarrow x = 2$

$3x - 1 = -5 \Rightarrow 3x = -4 \Rightarrow x = -\frac{4}{3}$

$S = \{2; -\frac{4}{3}\}$

$(x+1)^2 = -4$
$(x+1)^2 \geq 0, (\forall) x \in \mathbb{R}$
$-4 < 0$

} imposibil $\Rightarrow S = \emptyset$

$\frac{1}{9}\cdot x^2+3=\frac{76}{25} \Leftrightarrow \frac{1}{9}\cdot x^2=\frac{76}{25}-\frac{3}{1} \Leftrightarrow \frac{1}{9}\cdot x^2=\frac{76}{25}-\frac{75}{25} \Leftrightarrow \frac{1}{9}\cdot x^2=\frac{1}{25} \Leftrightarrow x^2=\frac{1}{25}:\frac{1}{9} \Leftrightarrow$

$x^2=\frac{1}{25}\cdot\frac{9}{1} \Leftrightarrow x^2=\frac{9}{25} \Leftrightarrow \sqrt{x^2}=\sqrt{\frac{9}{25}} \Leftrightarrow |x|=\frac{3}{5} \Rightarrow \begin{cases} x=\frac{3}{5} \\ \text{sau} \\ x=-\frac{3}{5} \end{cases} \Rightarrow S=\left\{\frac{3}{5};-\frac{3}{5}\right\}$

$\frac{1}{4}\cdot x^2+0{,}1=\sqrt{0{,}25} \Leftrightarrow \frac{1}{4}\cdot x^2+\frac{2}{5}=\frac{1}{2} \Leftrightarrow$

$0{,}1=\frac{1}{10}=\frac{2}{5}$

$\sqrt{0{,}25}=\sqrt{\frac{25}{100}}=\frac{\sqrt{25}}{\sqrt{100}}=\frac{5}{10}=\frac{1}{2}$

$\frac{1}{4}\cdot x^2=\frac{1}{2}-\frac{2}{5} \Leftrightarrow \frac{1}{4}\cdot x^2=\frac{5}{10}-\frac{4}{10} \Leftrightarrow \frac{1}{4}\cdot x^2=\frac{1}{10}$

$x^2=\frac{1}{10}:\frac{1}{4} \Leftrightarrow x^2=\frac{1}{10}\cdot\frac{4}{1} \Leftrightarrow x^2=\frac{4}{10} \Rightarrow x^2=\frac{2}{5} \Leftrightarrow \sqrt{x^2}=\sqrt{\frac{2}{5}} \Leftrightarrow |x|=\frac{\sqrt{2}}{\sqrt{5}}$

$x=\frac{\sqrt{10}}{5}$ sau $x=-\frac{\sqrt{10}}{5} \Rightarrow S=\left\{\frac{\sqrt{10}}{5};-\frac{\sqrt{10}}{5}\right\}$

$5\sqrt{3}=\sqrt{75}=\sqrt{100-25} \neq \sqrt{100}-\sqrt{25}=10-5<15$

NU ESTE ADEVĂRAT!

$x^2=100-25 \Rightarrow \sqrt{x^2}=\sqrt{100-25} \Leftrightarrow |x|=\sqrt{100}-\sqrt{25}$

Logrești

$x^2=100\cdot 25 \Leftrightarrow$

$\sqrt{x^2}=\sqrt{100\cdot 25} \Leftrightarrow$

$|x|=\sqrt{100}\cdot\sqrt{25} \Leftrightarrow$

$|x|=10\cdot 5 \Leftrightarrow$

$|x|=50 \Leftrightarrow$

$x=50$ sau $x=-50$

$S=\{50;-50\}$

$|x|=\sqrt{75} \Leftrightarrow |x|=5\sqrt{3} \Leftrightarrow \begin{cases} x=5\sqrt{3} \\ \text{sau} \\ x=-5\sqrt{3} \end{cases}$

$S=\{5\sqrt{3};-5\sqrt{3}\}$

75	5
15	5
3	3
1	

$75=5^2\cdot 3$ (5)

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