Calcule cu radicali

$k \in \mathbb{N}$

Dacă $a \in \mathbb{R} \Rightarrow \sqrt{a^2} = |a|$ $\quad \sqrt{a^{2k}} = \sqrt{(a^k)^2} = |a|^k$

Ex: $\sqrt{(-3)^2} = |-3| = 3$ $\quad \sqrt{(-5)^4} = \sqrt{(5^2)^2} = \sqrt{25^2} = 25$

$\sqrt{(-5)^4} = |-5|^2 = 5^2 = 25$

$$\boxed{\begin{array}{l}\text{Dacă } a \geq 0, b \geq 0 \Rightarrow \\ \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \\ \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad (b \neq 0) \\ \sqrt{a : b} = \sqrt{a} : \sqrt{b}\end{array}}$$

$\sqrt{2^{10}} = 2^5 = 32$

$\sqrt{2^8 \cdot 5^4 \cdot 3^6} = \sqrt{2^8} \cdot \sqrt{5^4} \cdot \sqrt{3^6} = 2^4 \cdot 5^2 \cdot 3^3 = 2^2 \cdot 5^2 \cdot 2^2 \cdot 27 =$

$= 100 \cdot 4 \cdot 27 = 10800$

$$\boxed{\begin{array}{l} a \geq 0 \\ \sqrt{a^{2k+1}} = \sqrt{a \cdot a^{2k}} = \sqrt{a} \cdot \sqrt{a^{2k}} \Rightarrow \\ \sqrt{a^{2k+1}} = a^k \sqrt{a} \end{array}}$$

Ex: $\sqrt{5^7} = \sqrt{5^6 \cdot 5^1} = \sqrt{5^6} \cdot \sqrt{5} = 5^3 \cdot \sqrt{5} = 125\sqrt{3}$

$\sqrt{2^7 \cdot 3^4} = \sqrt{2^6 \cdot 3^4 \cdot 2} = \sqrt{2^6} \cdot \sqrt{3^4} \cdot \sqrt{2} = 2^3 \cdot 3^2 \cdot \sqrt{2} = 8 \cdot 9 \sqrt{2} = 72$

$\sqrt{2^9 \cdot 3^4 \cdot 5^2} = \sqrt{2^8 \cdot 3^4 \cdot 5^2 \cdot 2} = 2^4 \cdot 3^2 \cdot 5^1 \cdot \sqrt{2} = 16 \cdot 9 \cdot 5 \cdot \sqrt{2} = 720\sqrt{2}$

$\underbrace{\sqrt{13} = \sqrt{4+9} = \sqrt{2^2 + 3^2}}_{\text{corect}} \neq \underbrace{\sqrt{2^2} + \sqrt{3^2} = 2 + 3 = 5}_{\text{corect}}$

$\underbrace{\sqrt{9} = \sqrt{25-16} = \sqrt{5^2 - 4^2}}_{\text{corect}} \neq \underbrace{\sqrt{5^2} - \sqrt{4^2} = 5 - 4 = 1}_{\text{corect}}$

$$\boxed{\begin{array}{l} \sqrt{a+b} \neq \sqrt{a} + \sqrt{b} \\ \sqrt{a-b} \neq \sqrt{a} - \sqrt{b} \end{array}}$$

$\sqrt{1024} = \sqrt{2^{10}} = 2^5 = 32$

$\sqrt{584} = ?$

$\sqrt{193} = ?$

$\underbrace{13 = 9 + 4 = 3^2 + 2^2}_{\text{corect}} \neq \underbrace{(3+2)^2 = 5^2 = 25}_{\text{corect}}$

$\underbrace{5=9-4=3^2-2^2}_{\text{corect}} \neq \underbrace{(3-2)^2=1^2=1}_{\text{corect}}$

$$\boxed{\begin{array}{l} a^2+b^2 \neq (a+b)^2 \\ a^2-b^2 \neq (a-b)^2 \end{array}}$$

$\sqrt{584}=\sqrt{2^3\cdot 73}=\sqrt{2^2\cdot 2\cdot 73}=2\sqrt{146}$

$$\begin{array}{r|l} 584 & 2 \\ 292 & 2 \\ 146 & 2 \\ 73 & 73 \\ 1 & \end{array}$$

$$\begin{array}{r|l} 193 & 193 \\ 1 & \end{array}$$

$\sqrt{193}=\sqrt{193}$

$2\nmid 193$, $3\nmid 193$, $5\nmid 193$, $7\nmid 193$, $11\nmid 193$, $13\nmid 193$

$193:7=27$, $7<27$

$$\begin{array}{l} 14 \\ \hline =53 \\ \;\;49 \\ \hline =4 \end{array}$$

$193:11=17$, $11<17$

$$\begin{array}{l} 11 \\ \hline \;83 \\ \;77 \\ \hline =6 \end{array}$$

$193:13=14$, $13<14$

$$\begin{array}{l} 13 \\ \hline =63 \\ \;\;52 \\ \hline \;\;\;9 \end{array}$$

$193:17=11$, $17<11$ (F)

$$\begin{array}{l} 17 \\ \hline =23 \\ \;\;17 \\ \hline \;\;\;6 \end{array}$$

$17\nmid 193$

$\Downarrow$

$193=$ nr. prim

Introducerea factorilor sub radical: $a\sqrt{b}=\sqrt{a^2\cdot b}$, $a\geq 0$, $b\geq 0$

Ex: $2\sqrt{3}=\sqrt{2^2\cdot 3}=\sqrt{4\cdot 3}=\sqrt{12}$

$3\sqrt{5}=\sqrt{9\cdot 5}=\sqrt{45}$; $7\sqrt{10}=\sqrt{49\cdot 10}=\sqrt{490}$; $13\sqrt{2}=\sqrt{169\cdot 2}=\sqrt{338}$

$\underbrace{-2}_{\leq 0}\sqrt{3}=\sqrt{(-2)^2\cdot 3}=\sqrt{4\cdot 3}=\sqrt{12}$ (FALS)

$\hookrightarrow \geq 0$

$2\sqrt{3}=\sqrt{4\cdot 3}=\sqrt{12}$

$-2\sqrt{3}=-\sqrt{2^2\cdot 3}=-\sqrt{4\cdot 3}=-\sqrt{12}$

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