$$(a+b)^2=a^2+2ab+b^2$$
$$(a-b)^2=a^2-2ab+b^2$$
$$(a+b)\cdot(a-b)=a^2-b^2$$

$$\frac{2\sqrt2-3}{2\sqrt2+3}=?$$

$2\sqrt2-3$

$$\frac{2\sqrt2-3}{2\sqrt2+3}=\frac{(2\sqrt2-3)^2}{(2\sqrt2+3)\cdot(2\sqrt2-3)}=\frac{(2\sqrt2)^2-2\cdot2\sqrt2\cdot3+3^2}{(2\sqrt2)^2-3^2}=\frac{8-12\sqrt3+9}{8-9}=\frac{17-12\sqrt3}{-1}=-\frac{(17-2\sqrt3)}{1}=$$
$$=-17+2\sqrt3=2\sqrt3-17$$

se amplifică cu conjugatul numitorului

$\sqrt a+\sqrt b \Rightarrow \sqrt a-\sqrt b$

$\sqrt a-\sqrt b \Rightarrow \sqrt a+\sqrt b$

numitorul — conjugatul

$2\sqrt6+1$

$$\frac{3\sqrt2-5}{2\sqrt6-1}=\frac{(2\sqrt6+1)\cdot(3\sqrt2-5)}{(2\sqrt6-1)\cdot(2\sqrt6+1)}=$$

$$\sqrt{12}=\sqrt{4\cdot3}=2\sqrt3$$

$$=\frac{6\cdot\sqrt{12}-10\sqrt6+3\sqrt2-5}{(2\sqrt6)^2-1^2}=\frac{6\cdot2\sqrt3-10\sqrt6+3\sqrt2-5}{24-1}=\frac{12\sqrt3-10\sqrt6+3\sqrt2-5}{23}$$

$$3\sqrt5\cdot\sqrt3=3\cdot\sqrt{15}$$

$2+\sqrt3$

$$\frac{3\sqrt5}{2-\sqrt3}=\frac{3\sqrt5\cdot(2+\sqrt3)}{2^2-(\sqrt3)^2}=\frac{6\sqrt5+3\sqrt{15}}{4-3}=\frac{6\sqrt5+3\sqrt{15}}{1}=6\sqrt5+3\sqrt{15}$$

$$(5\sqrt5)^2=5\sqrt5\cdot5\sqrt5=25\cdot5=125$$

$5\sqrt5-2$

$$\frac{5\sqrt5}{5\sqrt5+2}=\frac{5\sqrt5\cdot(5\sqrt5-2)}{(5\sqrt5)^2-2^2}=\frac{25\cdot5-10\sqrt5}{125-4}=\frac{125-10\sqrt5}{121}$$

$2\sqrt3+3\sqrt2$

$$\frac{3\sqrt6}{2\sqrt3-3\sqrt2}=\frac{3\sqrt6\cdot(2\sqrt3+3\sqrt2)}{(2\sqrt3)^2-(3\sqrt2)^2}=\frac{6\sqrt{18}+9\sqrt{12}}{12-18}=\frac{6\cdot3\sqrt2+9\cdot2\sqrt3}{-6}=\frac{18\sqrt2+18\sqrt3}{-6}=-\frac{\cancel{18}^{3}(\sqrt2+\sqrt3)}{\cancel{6}_{1}}$$
$$=-3\cdot(\sqrt2+\sqrt3)=-3\sqrt2-3\sqrt3$$

$2\sqrt3-3\sqrt2$

$$\frac{2\sqrt3-3\sqrt2}{2\sqrt3+3\sqrt2}=\frac{(2\sqrt3-3\sqrt2)^2}{(2\sqrt3)^2-(3\sqrt2)^2}=\frac{(2\sqrt3)^2-2\cdot2\sqrt3\cdot3\sqrt2+(3\sqrt2)^2}{12-18}=\frac{12-12\sqrt6+18}{-6}=-\frac{30-12\sqrt6}{6}=$$
$$=-\frac{\cancel{6}^{1}\cdot(5-2\sqrt6)}{\cancel{6}_{1}}=-5+2\sqrt6=2\sqrt6-5$$

$3\sqrt3+1$

$$\frac{3\sqrt3+1}{3\sqrt3-1}=\frac{(3\sqrt3+1)^2}{(3\sqrt3)^2-1^2}=\frac{(3\sqrt3)^2+2\cdot3\sqrt3\cdot1+1^2}{27-1}=\frac{27+6\sqrt3+1}{26}=\frac{28+6\sqrt3}{26}=\frac{\cancel{2}^{1}\cdot(14+3\sqrt3)}{\cancel{26}_{13}}=$$
$$=\frac{14+3\sqrt3}{13}$$

Formula radicalilor compuși:

$$\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A+\sqrt{A^2-B}}{2}} \pm \sqrt{\frac{A-\sqrt{A^2-B}}{2}}$$

$A \in \mathbb{R},\ B \in \mathbb{R},\ B \geq 0$

$A \geq \sqrt{B}$

$2\sqrt{6} = \sqrt{4}\cdot\sqrt{6} = \sqrt{24}$

Ex: (V1) $\sqrt{5-2\sqrt{6}} = \sqrt{5-\sqrt{24}} = \sqrt{\frac{5+\sqrt{25-24}}{2}} \pm \sqrt{\frac{5-\sqrt{25-24}}{2}} = \sqrt{\frac{5+\sqrt{1}}{2}} - \sqrt{\frac{5-\sqrt{1}}{2}} = \sqrt{\frac{6}{2}} - \sqrt{\frac{4}{2}} = \sqrt{3}-\sqrt{2}$

$a^2 \pm 2ab + b^2 = (a \pm b)^2$

$\sqrt{a^2} = |a|$

$\sqrt{(-3)^2} = |-3| = 3$

$|12| = 12$

$|-12| = -(-12) = 12$

(V2) $\sqrt{5-2\cdot\sqrt{6}} = \sqrt{(\sqrt{3})^2 - 2\sqrt{3}\cdot\sqrt{2} + (\sqrt{2})^2} = \sqrt{(\sqrt{3}-\sqrt{2})^2} = |\sqrt{3}-\sqrt{2}| = ?$

$3 > 2 \Rightarrow \sqrt{3} > \sqrt{2} \Rightarrow \sqrt{3}-\sqrt{2} > 0 \Rightarrow |\sqrt{3}-\sqrt{2}| = \sqrt{3}-\sqrt{2}$

$\sqrt{4+2\sqrt{3}} = \sqrt{4+\sqrt{12}} = \sqrt{\frac{4+\sqrt{16-12}}{2}} + \sqrt{\frac{4-\sqrt{16-12}}{2}} = \sqrt{\frac{4+\sqrt{4}}{2}} + \sqrt{\frac{4-\sqrt{4}}{2}} =$

$= \sqrt{\frac{4+2}{2}} + \sqrt{\frac{4-2}{2}} = \sqrt{3} + \sqrt{1} = \sqrt{3}+1$

$\sqrt{4+2\sqrt{3}} = \sqrt{3+2\cdot\sqrt{3}\cdot 1+1} = \sqrt{(\sqrt{3})^2 + 2\cdot\sqrt{3}\cdot 1 + 1^2} = \sqrt{(\sqrt{3}+1)^2} = |\sqrt{3}+1| = ?$

$\left.\begin{array}{l}\sqrt{3} > 0 \\ \sqrt{1} > 0\end{array}\right| \Rightarrow \sqrt{3}+1 > 0 \Rightarrow |\sqrt{3}+1| = \sqrt{3}+1$

$\sqrt{37-20\sqrt{3}} = \sqrt{37-\sqrt{1200}} = \sqrt{\frac{37+\sqrt{1369-1200}}{2}} - \sqrt{\frac{37-\sqrt{1369-1200}}{2}} =$

$= \sqrt{\frac{37+\sqrt{169}}{2}} - \sqrt{\frac{37-\sqrt{169}}{2}} = \sqrt{\frac{37+13}{2}} - \sqrt{\frac{37-13}{2}} = \sqrt{\frac{50}{2}} - \sqrt{\frac{24}{12}} = \sqrt{25} - \sqrt{12} =$

$= 5 - \sqrt{4\cdot 3} = 5 - 2\sqrt{3}$

$\sqrt{37-20\sqrt{3}} = \sqrt{(2\sqrt{3})^2 - 2\cdot 5\cdot 2\sqrt{3} + 5^2} = \sqrt{(2\sqrt{3}-5)^2} = |2\sqrt{3}-5| = ?$

$\left.\begin{array}{l}2\sqrt{3} = \sqrt{4}\cdot\sqrt{3} = \sqrt{4\cdot 3} = \sqrt{12} \\ 5 = \sqrt{25}\end{array}\right\} 12 < 15 \Rightarrow \sqrt{12} < \sqrt{15} \Rightarrow 2\sqrt{3} < 5 \Rightarrow 2\sqrt{3}-5 < 0$

$|2\sqrt{3}-5| = -(2\sqrt{3}-5) = -2\sqrt{3}+5 = 5-2\sqrt{3}$

$$\sqrt{9-4\sqrt5} = ?$$

$$\sqrt{9-4\sqrt5} = \sqrt{9-\sqrt{80}} = \sqrt{\frac{9+\sqrt{81-80}}{2}} - \sqrt{\frac{9-\sqrt{81-80}}{2}} = \sqrt{\frac{9+1}{2}} - \sqrt{\frac{9-1}{2}} = \sqrt5-\sqrt4 = \sqrt5-2$$

$$\sqrt{9-4\sqrt5} = \sqrt{\underbrace{2^2}_{4} - 2\cdot2\sqrt5 + \underbrace{(\sqrt5)^2}_{5}} = \sqrt{(2-\sqrt5)^2} = |2-\sqrt5| = ?$$

$2=\sqrt4$

$4<5 \Rightarrow \sqrt4<\sqrt5 \Rightarrow 2-\sqrt5<0 \Rightarrow |2-\sqrt5| = -(2-\sqrt5) = -2+\sqrt5 = \sqrt5-2$

....